PROJECTIONS FOR WORLD MAPS [1]

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The Conformal Variant of The Butterfly Map of the World [2] is now, after 25 years' study, perfected for the use of meteorologists. The author firmly believes that no better world map is possible for this purpose.

In designing a world map projection, as in designing a building, a just compromise must be harmoniously arrived at between various conflicting demands so that the more insistent shall prevail over the less insistent. Thus a disk on top of each pole would yield the maximum of arctic accuracy.

But if this disk is expanded to cover the hemisphere the Temperate Zones will not be so good and the equatorial regions will be worse. On the other hand, a cylindrical belt around the equatorial regions yields the maximum accuracy for the Torrid Zone. But if this belt be extended, the temperate regions are now not so good and the polar regions bad. Whereas, if, as in this map, we conceive the world projected conically—that is, from the Temperate Zone—why, then, upon extension, the polar and equatorial regions may not be so good, but no part of the world is downright bad as in both of the preceding methods.

It is to be noted that conformal projections on all three systems, resulting in five distinct maps (see fig. 1, not reproduced), were suggested by Prof. V. Bjerknes and illustrated in the "Geografiska Annaler." But, except for local intensive studies, the disruption of the land masses and the broken continuity of this 5-part system render it of little use for the presentation of phenomena which extend and move freely and impartially all over the globe.

It is quite obvious that the ideal single world map, to avoid extremes, must be based not on the horizontal plane of the polar disk, nor in the vertical plane of the equatorial cylinder, but in an oblique plane between these two extremes.

Nor may we adopt any conical system upon which the parallels are concentric rings around the poles, because in this case the Northern and Southern Hemispheres are bounded by a circular Equator and the world is cut in two parts with only rolling contact at a single point where line contact is most desirable even at the present time and sure to become vastly more so when the Southern Hemisphere extends its observatories and those important movements of the upper air across the Equator shall be graphically recorded and yield as a consequence their true meaning to the long-range forecaster.

It should now be clear why (1) the polar regions are shown in 240° of arc instead of 360, and why (2) the Equator is shown as four straight lines (two-thirds of a hexagon) instead of a complete circle, and why the parallels are interlinking ellipses which become straight lines at the Equator.

It must be remembered that this map is deliberately designed to be the most accurate possible of the whole world on a single diagram, and to this end it is planned around the actual contours of the earth so as to encompass the pattern of the continents and adjacent seas in the closest fitting frame possible. It is obvious that the human world spreads east and west in the Northern Hemisphere (if we include the great belts of transoceanic intercommunication). This, the main body of the map, takes in by far the major part of the earth's anthropogeographical interest. There remain the three land peninsulas radiating from the major mass and the island groups of the South Pacific. While the four southern octants can be arranged in a two-thirds hexagon exactly like the northern mass, when these are added one by one to the northern group to complete the radiating land extensions of South Africa, South America, Australia, and New Zealand they necessarily become separated. But considering the enormous disparity between the human importance of the northern nucleus compared to the southern streamers, this arrangement of the butterfly map in its discontinuous aspect is not only a fair price to pay for the great accuracy achieved by this sacrifice but it presents a logical pictorial expression of the world's ground plan, the home of the human race—a grand central Temperate Zone pavilion with four subordinate radiating wings which extend across the Equator toward the South Pole.

Let us now consider in detail to what precise extent the polar and equatorial regions of the butterfly map are not so good for the purposes of the meteorologists.

In the first place any world-map projection conceived conically, that is with the Temperate Zone in the most favorable position, must necessarily first be split up the back before it can be laid out in a fan-shaped plane. In the particular case of the conformal variant we substitute for a cone over each Hemisphere a demioctahedron or pyramid, the four faces of which are very logically equilateral triangles because the two side boundaries with the Equator at the base are all three just a quarter of the earth's circumference, or 90° by 90°—great circle degrees.

When the facets of such a pyramid are laid out in a plane they become exactly two-thirds of a hexagon, the gap or hiatus at the pole subtending therefore an angle of 120°. This arrangement is one of the fundamentals of the octahedral system of mapping the world because it permits the insertion in the polar gap of just two more facets or repeat octants which, when added to the eastern boundary, show in the completed hexagon an additional half of the hemisphere. Thus any forecaster can see spread before him 36 hours of recorded weather instead of 24.

Now for dynamic purposes (that is, showing constantly changing phenomena over the surface of a map rotating clockwise and capable of indefinite extension as time passes in a counterclockwise direction), this arrangement has obvious advantages over a static or closed system, since the story of the weather is told not in a single map, but in a succession of maps.

Since conformality is universally conceded to be a *sine qua non* of weather maps and since the very essence of conformality consists in a stretching or expansion of scale from a point or a line to the limits of the map, it is clear that variation of scale can not be avoided. And the larger the scope of the map the greater will be the expansion.

Whenever wind velocity is calculated from the relative closeness of isobars, it follows that formulas for this purpose depend on a considerable number of barometric readings, very few of which can be expected much above the eightieth parallel. A glance at the map will show that if we regard the 20° diameter polar cap as a sort of meteorological limbo where nothing much happens (or if things do happen they will not be observed), and therefore not recorded, it will be seen that we have considerably discounted the drawbacks of two of the six regions where the scale is most violently enlarged. As for the

[1] Presented before the meeting of the American Meteorological Society at New York Dec. 28, 1928.

[2] See Figures 1a and 1b, pp. 130-131.

remaining four points on the Equator where the scale reaches its maximum expansion and conformality fails, these are all in the acknowledged meteorological regions of least action, the doldrums, all well out to sea and in localities, on the whole, free from islands or even the tracks of sea-born trade. If they ever are active meteorologically they are in effect practically passive from want of recording stations either fixed or transient. And here let it be noted that in this rhombic type of map as compared with the conical type the general scale is *not* second-rate at the Equator except at the corners because the very best scale of the map is along the hexagonal mesh forming the perpendiculars to the three sides of each octant with the corners of the hexagons at the centers of the octants. This hexagonal mesh is marked on the map which shows that the three radiating continental plateaux and the Pacific Ocean in their north and south extension share the maximum scale accuracy along with the two Temperate Zones in their extensions east and west.

The seventy-odd great-circle degrees defining a side of this hexagon is in its total extent equal to the same interval along the surface of the globe of derivation. Although scale increase toward the extreme corners is largely mitigated in actual practice for the reasons above stated, it is put forward as a serious objection to the adoption of this map for the purposes of international meteorology in spite of the maps' general superiority in so many other particulars.

The objection is an unreasonble one because, as we have seen, any conformal map of the world *must* develop scale variation and if the same objection had been made to Mercator's chart, where the scale enlargement is infinitely greater, the progress of navigation would have been very seriously delayed.

It seems clear that the mathematically minded meteorologists who worked out the formulas for computing wind velocity from isobar gradients, etc., have considered the problem from an intensive small area, rather than from an extensive large-area viewpoint. Small-scope weather charts such as are used in northwestern Europe would not have to deal with the scale difficulties encountered in surveys of very much greater magnitude. Hence, their formulas would naturally assume the scale factor as a constant.

But now that the scope of meteorology is ever broadening to world wide dimensions, we find the experts, as so often happens, still using the same old formulas derived from past practice, and when confronted with a map literally covering the whole earth they find the assumption of a constant scale will not work. Now if by changing the factors in the formula, so that the scale became a variable, or by dividing the map into regions of progressive scale magnitude these intensive problems could be solved, would this not be wiser than total rejection of a master world map which has so many compensating advantages?

Nothing seems more reasonable than the essential oneness of the world from the meteorologist's viewpoint and several eminent practitioners of this most useful science have told me that a rational world map for their use was a much needed want, a veritable zeitbedurfniss. And this means that the ideal international world map for the study of long-range forecasting problems and the meteorology of the future must have complete trans-equatorial continuity.

I now ask the reader to examine carefully the outlines of this map noting the following items:

(1) Except at six points all parallels and the meridians cross at right angles, an essential of conformality which gives validity to wind roses or compass directions impossible on any map with an oblique graticule. This is extremely important, not only as regards wind direction of both the lower and the upper air, but in the angles formed with isobars and the true direction of isobars and the paths of LOWS or storm tracks and the direction of HIGHS when these are moving.

(2) The meridians are for the most part straight lines or very nearly straight lines.

(3) The parallels are in the main hardly distinguishable from arcs, and as they expand into interlinking ellipses they straighten out into four lines at the Equator so that perfect contact is secured between the north and south octants.

(4) The whole surface of the map like the world itself is a "continuum by replacement" meaning that the overprinted records of moving phenomena can be extended from one octant to another, east, west, north or south continuously.

(5) "Hexagons of strength" (that is, lines of the best scale) bisect the Temperate Zones east and west and the radiating continental plateaux north and south.

(6) The weak parts of the map are in the corners alike of zero human interest and minimum observatorial possibilities.

(7) While the scale increases radially from the center of each octant (where it is slightly subnormal) to its bounds, it is at any point the same in all directions and where at the corners the scale increases the scope diminishes.

(8) The form of the grid is rigidly fixed and unchangeable like Mercator's map no matter what the size or scale. Moreover the computed weave of the graticule in this conformal map lends itself especially to the mathematical side of meteorology in a manner impossible on an equal-area projection. On the other hand, quantitative problems covering, for example, the weight of the atmosphere over given areas, can be easily solved since the actual areas of every 5° by 5° square from the Equator to the pole will be given on each map.

No satisfactory world map is possible which includes both polar integrity and transequatorial continuity, and to this may be added the dictum that no adequate world map can be developed by accident or by the uncontrolled extension of any mathematical weave of coordinates. The creation of a master plan of the world is a matter of design first and foremost, the design of a frame within which intensive mathematical details shall be afterwards subordinated. And this is a task for which the architect is peculiarly fitted owing to his familiarity with form, his mastery of diagram, his training in compromise and his skill in graphic coordination and synthesis.

Any one with the most rudimentary knowledge of projection realizes that while it is comparatively easy to scheme out excellent representations of small areas of the globe up to the size even of the United States, it is a matter of extraordinary difficulty to map a hemisphere and practically impossible to show the whole world on one continuous sheet, that is to say, with the same general accuracy obtainable on a small area.

On the conformal butterfly map here shown, however, on account of its deliberately decentralized and articulated design it is possible to show any small country, no matter where, with ordinary atlaslike accuracy. Of large countries the same thing is also true. Suppose, for example, we select the map of the United States printed